Faculty of Science

Modular Tree Automata

Deriving Modular Recursion Schemes from Tree Automata

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Goals

Syntax-directed computations on ASTs

- program analysis
- complex program transformations
- compiler construction in general

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Embed the solution into Haskell.

How do we achieve these goals?

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Locality

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Contextuality

syntax-directed functions may depend on (the result of) others

- NB: This breaks locality and has to be carefully restricted!
- But it is convenient/necessary for
  - compositionality
  - expressivity

Locality

Tree automata

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$$f(q_1(x_1), q_2(x_2), \ldots, q_n(x_n)) \quad \longrightarrow \quad q(t[x_1, x_2, \ldots, x_n])$$

Compositionality

We shall compose tree automata along 3 different dimensions.

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sequential composition: a.k.a. deforestation

$$\mu\mathcal{F}_1 \xrightarrow{\quad [\![\mathcal{A}_1]\!] \quad} \mu\mathcal{F}_2 \xrightarrow{\quad [\![\mathcal{A}_2]\!] \quad} \mu\mathcal{F}_3$$

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input signature: the type of the AST

$[\![\mathcal{A}_1]\!] : \mu\mathcal{F} \to R$

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\end{array}
\quad \Longrightarrow \quad
[\![\mathcal{A}_1 + \mathcal{A}_2]\!] : \mu(\mathcal{F} + \mathcal{G}) \to R
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output type: tupling / product automaton construction

$$[\![\mathcal{A}_1]\!] : \mu\mathcal{F} \to R$$

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tupling / product automaton construction

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mutumorphisms / dependent product automata

$$\begin{array}{l} \mathcal{A}_1 : \qquad \mathcal{F} \to R \\ \mathcal{A}_2 : R \Rightarrow \mathcal{F} \to S \end{array}$$

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Outline

Terms in Haskell

Data types as fixed points of functors

$$\mathbf{data}\ \mathit{Term}\ f = \mathit{In}\ (f\ (\mathit{Term}\ f))$$

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$\textbf{data}\ \mathit{Term}\ f = \mathit{In}\ (f\ (\mathit{Term}\ f))$

Functors

$\textbf{class}\ \mathit{Functor}\ f\ \textbf{where}$

$\quad \mathit{fmap} :: (a \to b) \to f\ a \to f\ b$

Bottom-Up State Transitions in Haskell

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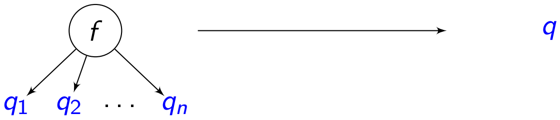
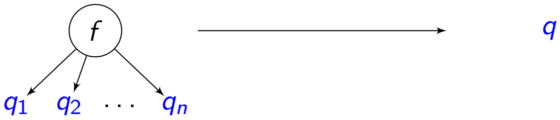


Bottom-Up State Transitions in Haskell



Bottom-up state transition rules as algebras

type $\mathit{UpState}\ f\ q = f\ q \to q$

Bottom-Up State Transitions in Haskell

Bottom-up state transition rules as algebras

type $UpState\ f\ q = f\ q \to q$

$$
\begin{aligned}
&runUpState \quad :: Functor\ f \Rightarrow UpState\ f\ q \to Term\ f \to q \\
&runUpState\ \phi\ (In\ t) = \phi\ (fmap\ (runUpState\ \phi)\ t)
\end{aligned}
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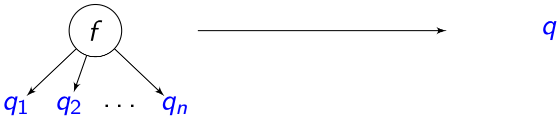
Bottom-Up State Transitions in Haskell



Bottom-up s … ras

a.k.a. catamorphism / fold

$\textbf{type}\ \mathit{UpState}\ f\ q = f\ q \rightarrow q$

$\mathit{runUpState} :: \mathit{Functor}\ f \Rightarrow \mathit{UpState}\ f\ q \rightarrow \mathit{Term}\ f \rightarrow q$
$\mathit{runUpState}\ \phi\ (\mathit{In}\ t) = \phi\ (\mathit{fmap}\ (\mathit{runUpState}\ \phi)\ t)$

Composing State Spaces – Motivating Example

A simple expression language

data $\mathit{Sig}\ e = \mathit{Val}\ \mathit{Int} \mid \mathit{Plus}\ e\ e$

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Task: writing a code generator

type $Addr = Int$
data $Instr\ = Acc\ Int \mid Load\ Addr \mid Store\ Addr \mid Add\ Addr$
type $Code = [Instr]$

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&\textbf{type}\ \mathit{Code} = [\mathit{Instr}]
\end{aligned}$$

The problem

$$\begin{aligned}
&\mathit{codeSt} :: \mathit{UpState}\ \mathit{Sig}\ \mathit{Code}\\
&\mathit{codeSt}\ (\mathit{Val}\ i) \qquad = [\mathit{Acc}\ i]\\
&\mathit{codeSt}\ (\mathit{Plus}\ x\ y) = x +\!\!+ [\mathit{Store}\ a] +\!\!+ y +\!\!+ [\mathit{Add}\ a]\\
&\quad \textbf{where}\ a = \ldots
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A simple expression language

data *Sig e* = *Val Int* | *Plus e e*

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type *Addr* = *Int*
data *Instr* = *Acc Int* | *Load Addr* | *Store Addr* | *Add Addr*
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$Sig\ Code \rightarrow Code$

The problem

codeSt :: *UpState Sig Code*
codeSt (*Val i*) = [*Acc i*]
codeSt (*Plus x y*) = *x* ++ [*Store a*] ++ *y* ++ [*Add a*]
  where *a* = . . .

Tupling

Tuple the code with an address counter

$$
\begin{aligned}
&\mathit{codeAddrSt} :: \mathit{UpState}\ \mathit{Sig}\ (\mathit{Code}, \mathit{Addr})\\
&\mathit{codeAddrSt}\ (\mathit{Val}\ i) && = ([\mathit{Acc}\ i], 0)\\
&\mathit{codeAddrSt}\ (\mathit{Plus}\ (x, a')\ (y, a)) && = (x +\!\!+ [\mathit{Store}\ a] +\!\!+ y +\!\!+ [\mathit{Add}\ a],\\
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Run the automaton

$$
\begin{aligned}
&\mathit{code} :: \mathit{Term}\ \mathit{Sig} \rightarrow (\mathit{Code}, \mathit{Addr})\\
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Product Automata

Deriving projections

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Product state transition

$$(\otimes) :: (p \in c, q \in c) \Rightarrow DUpState\ f\ c\ p \to DUpState\ f\ c\ q \to DUpState\ f\ c\ (p, q)$$

$$(sp \otimes sq)\ t = (sp\ t, sq\ t)$$

Running Dependent State Transition Functions

The types

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&\textbf{type}\ \mathit{UpState}\ \ f\ q &&= &&f\ q \to q \\
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Running Dependent State Transition Functions

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$$\textbf{type } \mathit{UpState} \quad f \; q \quad = \qquad\qquad f \; q \to q$$
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Running dependent state transitions

$$\mathit{runDUpState} :: \mathit{Functor} \; f \Rightarrow \mathit{DUpState} \; f \; q \; q \to \mathit{Term} \; f \to q$$
$$\mathit{runDUpState} \; f = \mathit{runUpState} \; f$$

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From state transition to dependent state transition

$$\mathit{dUpState} :: \mathit{Functor}\ f \Rightarrow \mathit{UpState}\ f\ q \to \mathit{DUpState}\ f\ p\ q$$
$$\mathit{dUpState}\ \mathit{st} = \mathit{st}\ .\ \mathit{fmap}\ \mathit{pr}$$

The Code Generator Example

The code generator

$$\begin{aligned}
&\mathit{codeSt} :: (\mathit{Int} \in q) \Rightarrow \mathit{DUpState}\ \mathit{Sig}\ q\ \mathit{Code}\\
&\mathit{codeSt}\ (\mathit{Val}\ i) \qquad = [\mathit{Acc}\ i]\\
&\mathit{codeSt}\ (\mathit{Plus}\ x\ y) = \mathit{pr}\ x +\!\!+ [\mathit{Store}\ a] +\!\!+ \mathit{pr}\ y +\!\!+ [\mathit{Add}\ a]\\
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Generating fresh addresses

$$\begin{aligned}
&\mathit{heightSt} :: \mathit{UpState}\ \mathit{Sig}\ \mathit{Int}\\
&\mathit{heightSt}\ (\mathit{Val}\ _) \qquad = 0\\
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Combining the components

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&\mathit{code} :: \mathit{Term}\ \mathit{Sig} \rightarrow \mathit{Code}\\
&\mathit{code} = \mathit{fst}\ .\ \mathit{runUpState}\ (\mathit{codeSt} \otimes \mathit{dUpState}\ \mathit{heightSt})
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&\mathit{code} :: \mathit{Term}\ \mathit{Sig} \to \mathit{Code}\\
&\mathit{code} = \mathit{fst}\ .\ \mathit{runUpState}\ \overbrace{(\mathit{codeSt} \otimes \mathit{dUpState}\ \mathit{heightSt})}^{(\mathit{Code} \in q,\, \mathit{Int} \in q) \Rightarrow \mathit{DUpState}\ \mathit{Sig}\ q\ (\mathit{Code}, \mathit{Int})}
\end{aligned}
$$

Outline

Combining Signatures

Signatures & automata may be combined in the style of "Data types à la carte" [Swierstra 2008].

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Coproduct of signatures

$\textbf{data}\ (f \oplus g)\ e = \mathit{Inl}\ (f\ \ e) \mid \mathit{Inr}\ (g\ \ e)$

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Example

$\textbf{data}\ \mathit{Inc}\ e = \mathit{Inc}\ e$
$\textbf{type}\ \mathit{Sig}' \ \ = \mathit{Inc} \oplus \mathit{Sig}$

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Coproduct of signatures

$\textbf{data}\ (f \oplus g)\ e = \mathit{Inl}\ (f\ \ e) \mid \mathit{Inr}\ (g\ \ e)$

Example

$\textbf{data}\ \mathit{Inc}\ e = \mathit{Inc}\ e$
$\textbf{type}\ \mathit{Sig}' \ \ = \mathit{Inc} \oplus \mathit{Sig}$

Subsignature type class

$\textbf{class}\ f \preceq g\ \textbf{where}$
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$f \preceq g \quad$ iff

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- $f = g_i, \quad 0 < i \leq n$

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For example: $\mathit{Inc} \preceq \mathit{Sig}'$

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Combining Automata

Making the height compositional

class *HeightSt f* **where**
  $heightSt :: DUpState\ f\ q\ Int$

instance $(HeightSt\ f, HeightSt\ g) \Rightarrow HeightSt\ (f \oplus g)$ **where**
  $heightSt\ (Inl\ x) = heightSt\ x$
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Defining the height on Sig

$$\textbf{instance}\ \mathit{HeightSt}\ \mathit{Sig}\ \textbf{where}$$
$$\quad \mathit{heightSt}\ (\mathit{Val}\ \_) = 0$$
$$\quad \mathit{heightSt}\ (\mathit{Plus}\ x\ y) = 1 + \mathit{max}\ x\ y$$

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instance *HeightSt Sig* **where**
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$\quad heightSt\ (Plus\ x\ y) = 1 + max\ x\ y$

Defining the height on Inc

instance *HeightSt Inc* **where**
$\quad heightSt\ (Inc\ x) = 1 + x$

Outline

Bottom-Up Tree Transducers

Bottom-Up Tree Transducers

Bottom-Up Tree Transducers

From terms to contexts

$$\textbf{data}\ \ Term\ \ f\ \ = In\,(f\,(Term\ \ f\ \ ))$$
$$\textbf{data}\ Context\ f\ a = In\,(f\,(Context\ f\ a))\mid Hole\ a$$

Bottom-Up Tree Transducers

type *Term f* = *Context f Empty*

From terms to contexts

data *Term f* = *In* (*f* (*Term f*))

data *Context f a* = *In* (*f* (*Context f a*)) | *Hole a*

Bottom-Up Tree Transducers

From terms to contexts

$\textbf{data}\ \ Term\ \ f\ \ = In\,(f\,(Term\ \ f\ \ ))$

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Representing transduction rules, [Hasuo et al. 2007]

$\textbf{type}\ UpTrans\ f\ q\ g = \forall\ a.f\ (q,a) \to (q,\ Context\ g\ a)$

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Tree Homomorphisms

$$\textbf{type}\ \mathit{UpTrans}\ f\ q\ g = \forall\, a\, .\, f\ (q, a) \rightarrow (q, \mathit{Context}\ g\ a)$$

Tree Homomorphisms

$$\textbf{type}\ \mathit{UpTrans}\ f\quad g = \forall\, a\,.\ f\quad a \ \rightarrow\quad \mathit{Context}\ g\ a$$

Tree Homomorphisms

$$\textbf{type}\ \mathit{Hom}\ f\ g = \forall\, a\,.\, f\ a \rightarrow \mathit{Context}\ g\ a$$

Tree Homomorphisms

$$\textbf{type } \mathit{Hom}\ f\ g = \forall\, a\,.\, f\ a \rightarrow \mathit{Context}\ g\ a$$

Example (Desugaring)

$$\textbf{class } \mathit{DesugHom}\ f\ g \textbf{ where}$$
$$\quad \mathit{desugHom} :: \mathit{Hom}\ f\ g$$

$$\mathit{desugar} :: (\mathit{Functor}\ f, \mathit{Functor}\ g, \mathit{DesugHom}\ f\ g) \Rightarrow \mathit{Term}\ f \rightarrow \mathit{Term}\ g$$
$$\mathit{desugar} = \mathit{runHom}\ \mathit{desugHom}$$

Tree Homomorphisms

type $Hom\ f\ g = \forall\, a\, .\, f\ a \to Context\ g\ a$

Example (Desugaring)

class $DesugHom\ f\ g$ **where**
  $desugHom :: Hom\ f\ g$

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$desugar = runHom\ desugHom$

instance $(Sig \preceq g) \Rightarrow DesugHom\ Inc\ g$ **where**
  $desugHom\ (Inc\ x) = Hole\ x\ \text{`}plus\text{`}\ val\ 1$

instance $(Functor\ g, f \preceq g) \Rightarrow DesugHom\ f\ g$ **where**
  $desugHom = simpCxt\ .\ inj$

Tree Homomorphisms

type $Hom\ \ f\ \ g = \forall\, a\,.\, f\ \ a \rightarrow\ \ Context\ g\ a$

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class $DesugHom\ f\ g$ **where**
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$desugar :: (Functor\ f, Functor\ g, DesugHom\ f\ g) \Rightarrow Term\ f \rightarrow Term\ g$
$desugar = runHom\ desugHom$

instance $(Sig \preceq g) \Rightarrow$
  $desugHom\ (Inc\ x)$

$simpCxt :: Functor\ g \Rightarrow g\ a \rightarrow Context\ g\ a$
$simpCxt\ t = In\ (fmap\ Hole\ t)$

instance $(Functor\ g, f \preceq g) \Rightarrow DesugHom\ f\ g$ **where**
  $desugHom = \ simpCxt\ \ .\, inj$

Stateful Tree Homomorphisms

Decomposing tree transducers

type $Hom \; f \; g = \forall a \, . \, f \; a \rightarrow Context \; g \; a$
type $UpState \; f \; q = f \; q \rightarrow q$
type $UpTrans \; f \; q \; g = \forall a \, . \, f \; (q, a) \rightarrow (q, Context \; g \; a)$

Stateful Tree Homomorphisms

Decomposing tree transducers

$$\textbf{type}\ \mathit{Hom}\ f\ g = \forall\, a\,.\, f\ a \rightarrow \mathit{Context}\ g\ a$$
$$\textbf{type}\ \mathit{UpState}\ f\ q = f\ {\color{blue}q} \rightarrow {\color{blue}q}$$
$$\textbf{type}\ \mathit{UpTrans}\ f\ q\ g = \forall\, a\,.\, f\ ({\color{blue}q}, a) \rightarrow ({\color{blue}q}, \mathit{Context}\ g\ a)$$

Making homomorphisms dependent on a state

$$\textbf{type}\ \mathit{QHom}\ f\ q\ g = \forall\, a.\ f\ a \rightarrow \mathit{Context}\ g\ a$$

Stateful Tree Homomorphisms

Decomposing tree transducers

$$
\begin{aligned}
&\textbf{type}\ \mathit{Hom}\ f\ g = \forall a\,.\, f\ a \rightarrow \mathit{Context}\ g\ a \\
&\textbf{type}\ \mathit{UpState}\ f\ q = f\ q \rightarrow q \\
&\textbf{type}\ \mathit{UpTrans}\ f\ q\ g = \forall a\,.\, f\ (q, a) \rightarrow (q, \mathit{Context}\ g\ a)
\end{aligned}
$$

Making homomorphisms dependent on a state

$$\textbf{type}\ \mathit{QHom}\ f\ q\ g = \forall a.\ f(q,\ a) \rightarrow \mathit{Context}\ g\ a$$

Stateful Tree Homomorphisms

Decomposing tree transducers

$$\begin{array}{l} \textbf{type } \mathit{Hom}\ f\ g = \forall\, a\,.\, f\ a \to \mathit{Context}\ g\ a \\ \textbf{type } \mathit{UpState}\ f\ q = f\ q \to q \\ \textbf{type } \mathit{UpTrans}\ f\ q\ g = \forall\, a\,.\, f\ (q, a) \to (q, \mathit{Context}\ g\ a) \end{array}$$

Making homomorphisms dependent on a state

$$\textbf{type } \mathit{QHom}\ f\ q\ g = \forall\, a.\ (a \to q) \to f\ a \to \mathit{Context}\ g\ a$$

Stateful Tree Homomorphisms

Decomposing tree transducers

$$
\begin{array}{l}
\textbf{type } \mathit{Hom} \; f \; g = \forall\, a\, .\, f \; a \to \mathit{Context}\; g\; a \\
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\textbf{type } \mathit{UpTrans}\; f\; q\; g = \forall\, a\, .\, f\; (q, a) \to (q, \mathit{Context}\; g\; a)
\end{array}
$$

Making homomorphisms dependent on a state

$$
\textbf{type } \mathit{QHom}\; f\; q\; g = \forall\, a.\, (a \to q) \to f\; a \to \mathit{Context}\; g\; a
$$

From stateful homomorphisms to tree transducers

$$
\begin{array}{l}
\mathit{upTrans} :: (\mathit{Functor}\; f, \mathit{Functor}\; g) \Rightarrow \\
\qquad\qquad \mathit{UpState}\; f\; q \to \mathit{QHom}\; f\; q\; g \to \mathit{UpTrans}\; f\; q\; g \\
\mathit{upTrans}\; \mathit{st}\; \mathit{hom}\; t = (q, c)\; \textbf{where} \\
\quad q = \mathit{st}\; (\mathit{fmap}\; \mathit{fst}\; t) \\
\quad c = \mathit{fmap}\; \mathit{snd}\; (\mathit{hom}\; \mathit{fst}\; t)
\end{array}
$$

An Example

Extending the signature with let bindings

$$\begin{aligned}
&\textbf{type } \mathit{Name} &&= \mathit{String}\\
&\textbf{data } \mathit{Let}\ e &&= \mathit{LetIn}\ \mathit{Name}\ e\ e \mid \mathit{Var}\ \mathit{Name}\\
&\textbf{type } \mathit{LetSig} &&= \mathit{Let} \oplus \mathit{Sig}
\end{aligned}$$

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$\textbf{type}\ \mathit{Vars} = \mathit{Set}\ \mathit{Name}$

$\textbf{class}\ \mathit{FreeVarsSt}\ f\ \textbf{where}$
$\quad \mathit{freeVarsSt} :: \mathit{UpState}\ f\ \mathit{Vars}$

An Example

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$$
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$$
\begin{aligned}
&\textbf{type } \mathit{Vars} = \mathit{Set}\ \mathit{Name}\\
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&\textbf{instance } \mathit{FreeVarsSt}\ \mathit{Sig}\ \textbf{where}\\
&\quad \mathit{freeVarsSt}\ (\mathit{Plus}\ x\ y) = x\ \text{`}\mathit{union}\text{`}\ y\\
&\quad \mathit{freeVarsSt}\ (\mathit{Val}\ \_) \quad = \mathit{empty}\\
&\textbf{instance } \mathit{FreeVarsSt}\ \mathit{Let}\ \textbf{where}\\
&\quad \mathit{freeVarsSt}\ (\mathit{Var}\ v) \qquad = \mathit{singleton}\ v\\
&\quad \mathit{freeVarsSt}\ (\mathit{LetIn}\ v\ e\ s) = \textbf{if } v\ \text{`}\mathit{member}\text{`}\ s\ \textbf{then } e\ \text{`}\mathit{union}\text{`}\ \mathit{delete}\ v\ s\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \textbf{else } s
\end{aligned}
$$

An Example (Cont'd)

class *RemLetHom f q g* **where**
  $remLetHom :: QHom\ f\ q\ g$

instance $(Vars \in q, Let \preceq g, Functor\ g) \Rightarrow RemLetHom\ Let\ q\ g$ **where**
  $remLetHom\ qOf\ (LetIn\ v\ \_\ s) \mid \neg\ (v\ `member`\ qOf\ s) = Hole\ s$
  $remLetHom\ \_\quad t = simpCxt\ (inj\ t)$

instance $(Functor\ f, Functor\ g, f \preceq g) \Rightarrow RemLetHom\ f\ q\ g$ **where**
  $remLetHom\ \_ = simpCxt\ .\ inj$

An Example (Cont'd)

$$
\begin{array}{l}
\textbf{class } \mathit{RemLetHom}\ f\ q\ g\ \textbf{where} \\
\quad \mathit{remLetHom} :: \mathit{QHom}\ f\ q\ g \\
\textbf{instance } (\mathit{Vars} \in q, \mathit{Let} \preceq g, \mathit{Functor}\ g) \Rightarrow \mathit{RemLetHom}\ \mathit{Let}\ q\ g\ \textbf{where} \\
\quad \mathit{remLetHom}\ \mathit{qOf}\ (\mathit{LetIn}\ v\ \_\ s) \mid \neg\, (v\ \text{`}\mathit{member}\text{`}\ \mathit{qOf}\ s) = \mathit{Hole}\ s \\
\quad \mathit{remLetHom}\ \_\ \quad t \qquad\qquad = \mathit{simpCxt}\ (\mathit{inj}\ t) \\
\textbf{instance } (\mathit{Functor}\ f, \mathit{Functor}\ g, f \preceq g) \Rightarrow \mathit{RemLetHom}\ f\ q\ g\ \textbf{where} \\
\quad \mathit{remLetHom}\ \_ = \mathit{simpCxt}\ .\ \mathit{inj}
\end{array}
$$

Combining state transition and homomorphism

$$
\begin{array}{l}
\mathit{remLet} :: (\mathit{Functor}\ f, \mathit{FreeVarsSt}\ f, \mathit{RemLetHom}\ f\ \mathit{Vars}\ f) \\
\qquad \Rightarrow \mathit{Term}\ f \to (\mathit{Vars}, \mathit{Term}\ f) \\
\mathit{remLet} = \mathit{runUpHom}\ \ \mathit{freeVarsSt}\ \mathit{remLetHom}
\end{array}
$$

An Example (Cont'd)

$$
\begin{array}{l}
\textbf{class } \mathit{RemLetHom}\ f\ q\ g\ \textbf{where} \\
\quad \mathit{remLetHom} :: \mathit{QHom}\ f\ q\ g \\
\textbf{instance } (\mathit{Vars} \in q, \mathit{Let} \preceq g, \mathit{Functor}\ g) \Rightarrow \mathit{RemLetHom}\ \mathit{Let}\ q\ g\ \textbf{where} \\
\quad \mathit{remLetHom}\ \mathit{qOf}\ (\mathit{LetIn}\ v\ \_\ s) \mid \neg\,(v\ \text{`}\mathit{member}\text{`}\ \mathit{qOf}\ s) = \mathit{Hole}\ s \\
\quad \mathit{remLetHom}\ \_\ \quad t \qquad\qquad\qquad\qquad\qquad\qquad = \mathit{simpCxt}\ (\mathit{inj}\ t) \\
\textbf{instance } (\mathit{Functor}\ f, \mathit{Functor}\ g, f \preceq g) \Rightarrow \mathit{RemLetHom}\ f\ q\ g\ \textbf{where} \\
\quad \mathit{remLetHom}\ \_
\end{array}
$$

$$
\begin{array}{l}
\mathit{runUpHom} :: \mathit{UpState}\ f\ q \to \mathit{QHom}\ f\ q\ g \\
\qquad\qquad \to \mathit{Term}\ f \to \mathit{Term}\ g \\
\mathit{runUpHom}\ \mathit{st}\ \mathit{hom} = \mathit{runUpTrans}\ (\mathit{upTrans}\ \mathit{st}\ \mathit{hom})
\end{array}
$$

Combining state

$$
\begin{array}{l}
\mathit{remLet} :: (\mathit{Func} \\
\qquad \Rightarrow \mathit{Term}\ f \to (\mathit{Vars}, \mathit{Term}\ f) \\
\mathit{remLet} = \mathit{runUpHom}\ \ \mathit{freeVarsSt}\ \mathit{remLetHom}
\end{array}
$$

An Example (Cont'd)

$$\textbf{class } \mathit{RemLetHom}\ f\ q\ g\ \textbf{where}$$
$$\quad \mathit{remLetHom} :: \mathit{QHom}\ f\ q\ g$$

$$\textbf{instance } (\mathit{Vars} \in q, \mathit{Let} \preceq g, \mathit{Functor}\ g) \Rightarrow \mathit{RemLetHom}\ \mathit{Let}\ q\ g\ \textbf{where}$$
$$\quad \mathit{remLetHom}\ \mathit{qOf}\ (\mathit{LetIn}\ v\ \_\ s) \mid \neg\,(v\ \text{`}\mathit{member}\text{`}\ \mathit{qOf}\ s) = \mathit{Hole}\ s$$
$$\quad \mathit{remLetHom}\ \_\ \quad t \qquad = \mathit{simpCxt}\ (\mathit{inj}\ t)$$

$$\textbf{instance } (\mathit{Functor}\ f, \mathit{Functor}\ g, f \preceq g) \Rightarrow \mathit{RemLetHom}\ f\ q\ g\ \textbf{where}$$
$$\quad \mathit{remLetHom}\ \_ = \mathit{simpCxt} \,.\, \mathit{inj}$$

Combining state transition and homomorphism

$$\mathit{remLet} :: (\mathit{Functor}\ f, \mathit{FreeVarsSt}\ f, \mathit{RemLetHom}\ f\ \mathit{Vars}\ f)$$
$$\qquad \Rightarrow \mathit{Term}\ f \to (\mathit{Vars}, \mathit{Term}\ f)$$
$$\mathit{remLet} = \mathit{runUpHom}\ \ \mathit{freeVarsSt}\ \mathit{remLetHom}$$

$$\mathit{remLet} :: \mathit{Term}\ \mathit{LetSig} \to \mathit{Term}\ \mathit{LetSig}$$
$$\mathit{remLet} :: \mathit{Term}\ (\mathit{Inc} \oplus \mathit{LetSig}) \to \mathit{Term}\ (\mathit{Inc} \oplus \mathit{LetSig})$$

Beyond Bottom-Up Tree Automata

What have we seen?

- Bottom-up tree acceptors (a.k.a. folds)
- Bottom-up tree transducers
- “dependent” versions thereof

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Other Tree recursion schemes

- Top-down tree acceptors
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- Bottom-up tree transducers
- "dependent" versions thereof

Other Tree recursion schemes

- Top-down tree acceptors
- Top-down tree transducers
- "dependent" versions thereof
- automata with bidirectional state propagation
- (restricted versions of macro tree transducers)

What have we gained?

Modularity & Reusability

- modularity along three dimensions (signature, sequential composition, state space)
- decoupling of state propagation and tree transformation
- operations on automata (beyond product & sum) allow us to construct new automata from old ones

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Modularity & Reusability

- modularity along three dimensions (signature, sequential composition, state space)
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- operations on automata (beyond product & sum) allow us to construct new automata from old ones

Interface between tree automata

- dependencies between automata by constraints on the state space
- modularity allows us to replace individual components

Try It Out!

This is part of the compositional data types Haskell library `compdata`:

```
> cabal install compdata
```

http://hackage.haskell.org/package/compdata