# Implementation of a Pragmatic Translation from Haskell into Isabelle/HOL

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# Outline

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# Haskell vs. Isabelle/HOL

## Haskell in a nutshell

- purely functional programming language
- non-strict semantics (mostly implemented by lazy evaluation)
- comprehensive type system: Hindley-Milner (restricted $F_\omega$) + type classes
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- HOL: Isabelle formulation of classical higher-order logic
- based on simply typed lambda calculus (system $F_1$)
  $\rightsquigarrow$ comparatively weak type system
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more details when we come to the implementation

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## Program verification

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## Example: l4.verified project

- aim: formalisation and verification of a microkernel
- prototype implementation in Haskell
- translation into Isabelle/HOL $\rightsquigarrow$ executable model
- reasoning about executable model in Isabelle/HOL

# Translation

## Goals

- cover a large subset of Haskell's syntax
- result should be easily readable
  - preserve syntactic structure as much as possible
  - translate syntactic sugar as well
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## Implementation

- implementation language: Haskell
- based on existing work from TU Munich

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# Haskell vs. Isabelle/HOL – Non-strictness/Partiality

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  $\leadsto$ recursive definitions need termination proof
- Haskell is Turing-complete $\leadsto$ partial functions definable
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Definitions that depend on non-strictness have to be avoided!

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**Example (Haskell)**

```
sumLen :: Int -> [a] -> [a] -> Int
sumLen s l1 l2 =  let len [] = 0
                      len (x:xs) = len xs + s
                  in len l1 + len l2
```

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## Example (Isabelle/HOL)

```
fun len1
where
  "len1 _ Nil = 0"
| "len1 s (x # xs) = len1 s xs + s"

fun sumLen :: "int => 'a list => 'a list => int"
where
  "sumLen s l1 l2 = (let len = len1 s
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```

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Our implementation is able to make these transformations!

# Haskell vs. Isabelle/HOL – Order of Definitions

- in Haskell definitions can appear in any order
- in Isabelle/HOL:
  - an identifier has to be defined before usage
  - mutual recursive definitions have to be made in parallel

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Our implementation reorders definitions accordingly!

# Haskell vs. Isabelle/HOL – Polymorphism

- Haskell: polymorphism over type constructors (of arbitrary kind)
- Isabelle/HOL: polymorphism over types only

## Example (type constructors)

- types (constructors of kind $*$): `Int`, `[Bool]`, `Int -> Bool`, ...
- type constructors of first-order kind: list (`[]`: $* \longrightarrow *$), sum (`Either`: $* \longrightarrow (* \longrightarrow *)$)
- type constructor of higher-order kind: `Tree`: $(* \longrightarrow *) \longrightarrow (* \longrightarrow *)$

```
data Tree c a = Node a (c (Tree c a))
```

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## Example (classes)

- type class:

```
class  (Eq a, Show a) => Num a  where
    (+), (-), (*)     :: a -> a -> a
    negate            :: a -> a
              ⋮
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- constructor class:

```
class  Monad m  where
    (>>=)  :: m a -> (a -> m b) -> m b
    return :: a -> m a
```

# Haskell vs. Isabelle/HOL – Ad Hoc Polymorphism II

- monad class is not definable in Isabelle/HOL!
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## Our solution

- Translate only instances of the class `Monad`!
- each monad instance has to use different names for the operation
  e.g. one monad uses $>>=$, return; another one uses $>>='$, return'
- type inference has to be performed to rename the operations correctly
- not full type inference is used, only a simple heuristics

# Haskell vs. Isabelle/HOL – Misc.

Further things that are taken care of in the translation

- as-patterns

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**Example**

In Haskell:

```
f :: [Int] -> [Int]
f l@(_:_) = 0 : l
f l@([])  = 1 : l
```

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## Example

In Haskell:

```
f :: [Int] -> [Int]
f l@(_:_) = 0 : l
f l@([])  = 1 : l
```

In Isabelle/HOL:

```
fun f where
  "f (a0 # a1)
      = (let l = (a0 # a1)
          in 0 # l)"
| "f Nil = (let l = Nil
              in 1 # l)"
```

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## Example (Haskell)

```
data MyRecord = A { aField1 :: String,
                    common1 :: Bool,
                    common2 :: Int }
              | B { bField1 :: Bool,
                    bField2 :: Int,
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              | C Bool Int String
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$\leadsto$ This is reduced to an ordinary data type!

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**Example (Haskell)**

```
insert :: Int -> [Int] -> [Int]
insert n [] = [n]
insert n (m:ms)
     | n < m     = n:m:ms
     | otherwise = m: insert n ms
```

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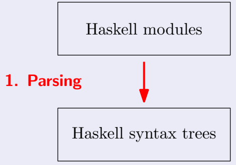
$\leadsto$ Guards are reduced to if-then-else expressions!

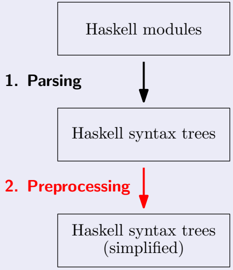
# Overall Design of Implementation – Parsing

- parse each Haskell module to a syntax tree
- imported modules are located and parsed as well
- parser only verifies context-free part of the syntax
- syntactically correct Haskell program is assumed



# Overall Design of Implementation – Preprocessing

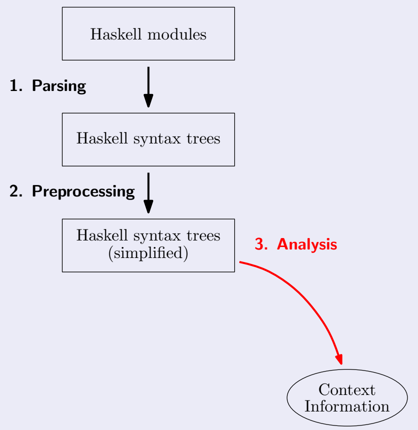
- Guards are transformed into if-then-else expressions.
- Local function definitions are transformed into top-level function definitions.
- Keywords and identifiers defined in the Isabelle/HOL library are renamed.



# Overall Design of Implementation – Analysis

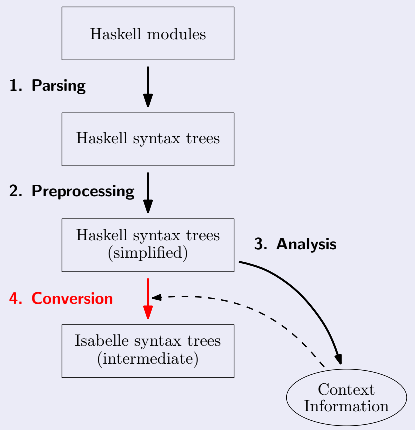
Some global information about the program is collected:

- type annotations
- the module where an identifier was defined
- what an identifier refers to (type, function etc.)
- associativity and precedence of defined operators



# Overall Design of Implementation – Conversion

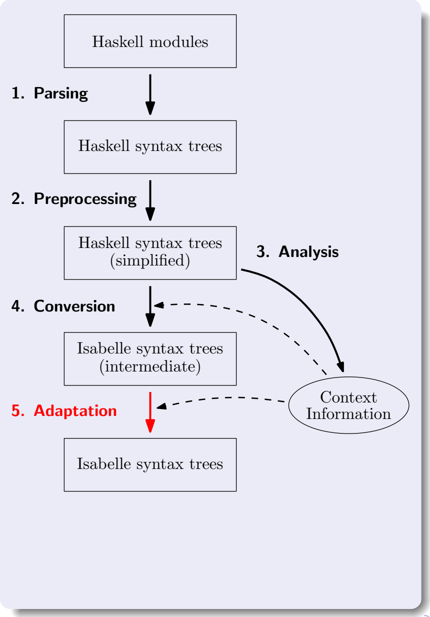
- Definitions are reordered according to their dependencies.
- Haskell syntax trees are translated into Isabelle/HOL syntax trees.



# Overall Design of Implementation – Adaptation

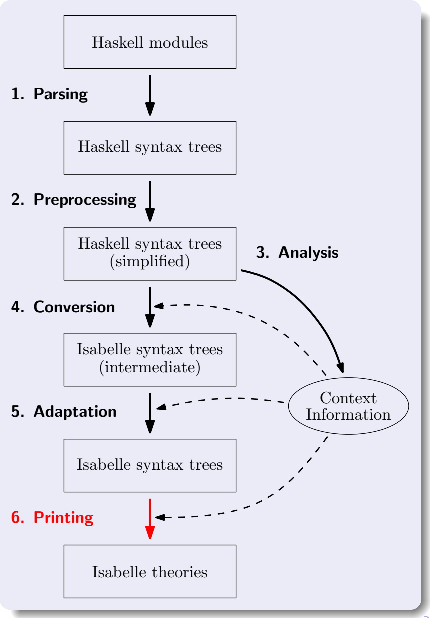
Renaming of predefined identifiers, e.g.:

- `Int` $\mapsto$ int
- `[]` $\mapsto$ Nil
- `++` $\mapsto$ `@`



# Overall Design of Implementation – Printing

- Isabelle/HOL syntax trees are written into theory files.



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# Summary

## Original implementation covered

- case, if-then-else, and let expressions
- list comprehensions
- where bindings
- as-patterns
- guards
- mutually recursive functions and data type definitions
- simple pattern bindings
- definitions and instantiations of type classes

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- ✘ as-patterns
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Some parts of the translations were unsound!

# Summary II

## Our Contributions

- ✔ mutually recursive function and data type definitions
- ✔ as-patterns
- ✔ guards
- data types with labelled fields
- closures in local function definitions
- monomorphic uses of monads

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## Our Contributions

- ✔ mutually recursive function and data type definitions
- ✔ as-patterns
- ✔ guards
- data types with labelled fields
- closures in local function definitions
- monomorphic uses of monads

## What is missing

- constructor type classes $\rightsquigarrow$ polymorphic uses of monads
- non-simple pattern bindings
- irrefutable patterns

# Conclusions

## What do we have

- translation is unsound!
- most of the Haskell 98 language can be translated
- resulting Isabelle/HOL formalisation is close to Haskell program
- comparatively easy reasoning in Isabelle/HOL
- adequate translation for most purposes $\rightsquigarrow$ l4.verified

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## Alternative Approach

- logic HOLCF is well suited to formalise partiality and non-strictness
- even constructor classes can be formalised
- reasoning in Isabelle/HOLCF is more complicated

# Coping with Large Data Types

Dealing with syntax trees $\Rightarrow$ dealing with large data types.

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- 500 lines of Haskell code
- 51 data types
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  ⇒ **Generic Programming + Code Generation**

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  ⇒ **Modularity**

# Generic Programming

"Scrap Your Boilerplate"

## Problem Addressed by SYB

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## Example

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- transform **`where`** clauses into **`let`** expressions

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- traverse a data structure to transform or query it
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### Example

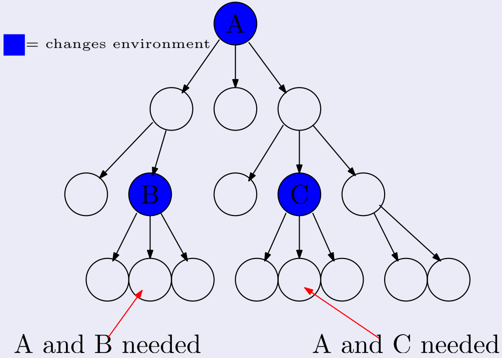
- compute free variables of an expression
- transform `where` clauses into `let` expressions

### Difficulties when Applying SYB in our Setting

- often context information is necessary
- We want to define a piece of context information only once.

# Environments

## Data Structure as a Tree



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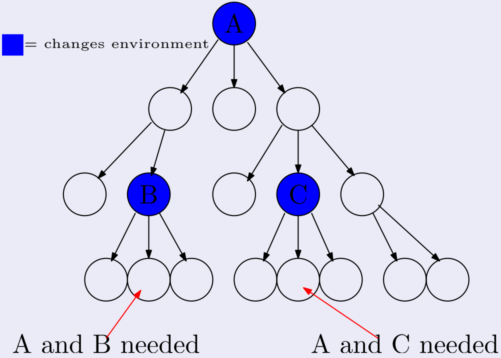


## Defining Environments by a -> (e -> e)

- a is the type of the current node
- e is the type of the environment

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## Generalisation of Environment Propagation

- non-uniform propagation
- monadic computations to define an environment