Generalising Tree Traversals to DAGs

Exploiting Sharing without the Pain

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Motivation

Goal

Do stuff on acyclic graphs, but pretend they are only trees.

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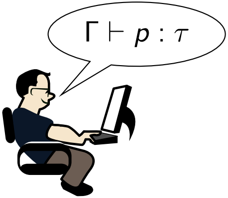
Primary Application

Abstract Syntax Graphs/Trees:

- type inference
- program analyses
- program transformations
- ...

The Idea



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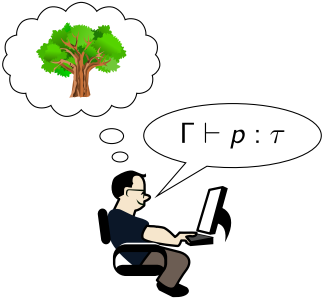

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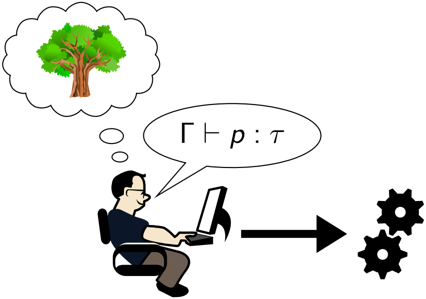

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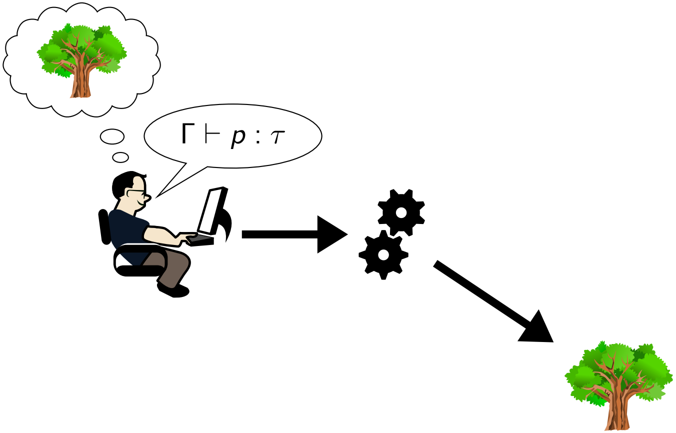

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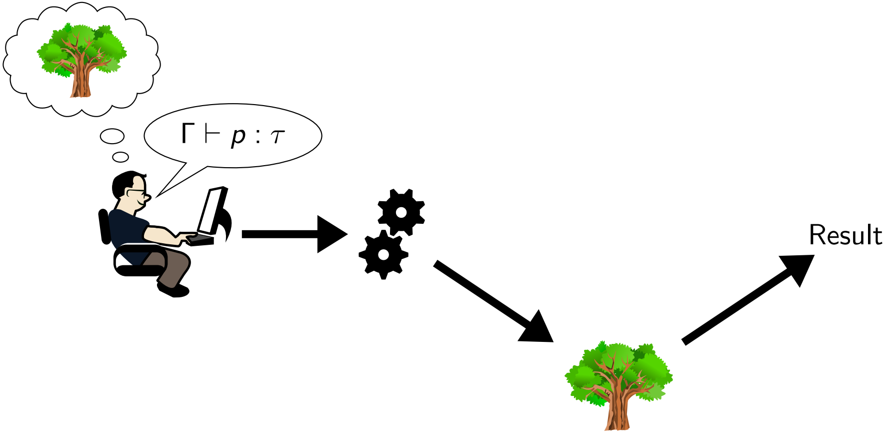

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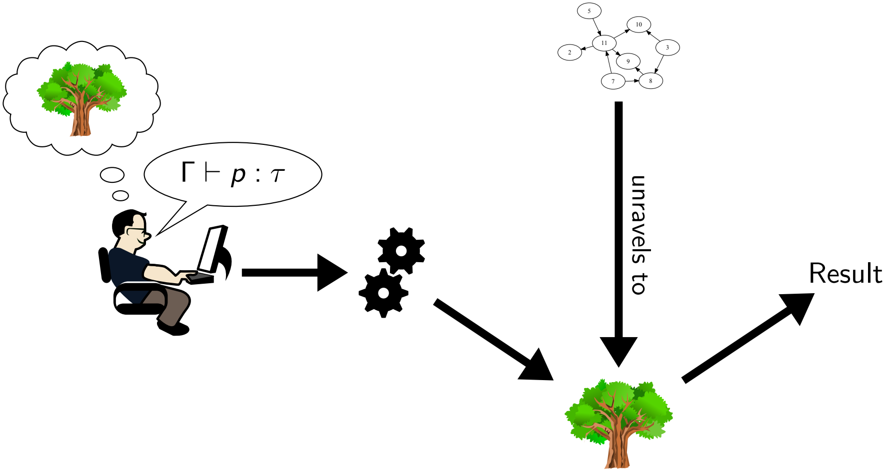

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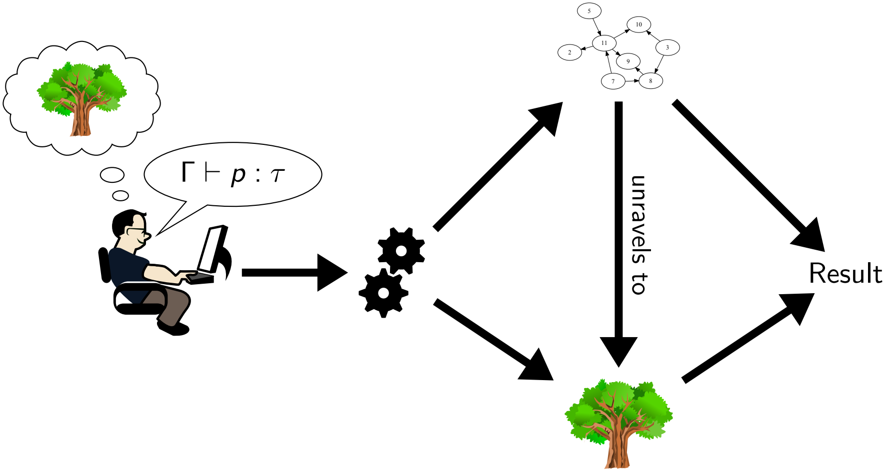

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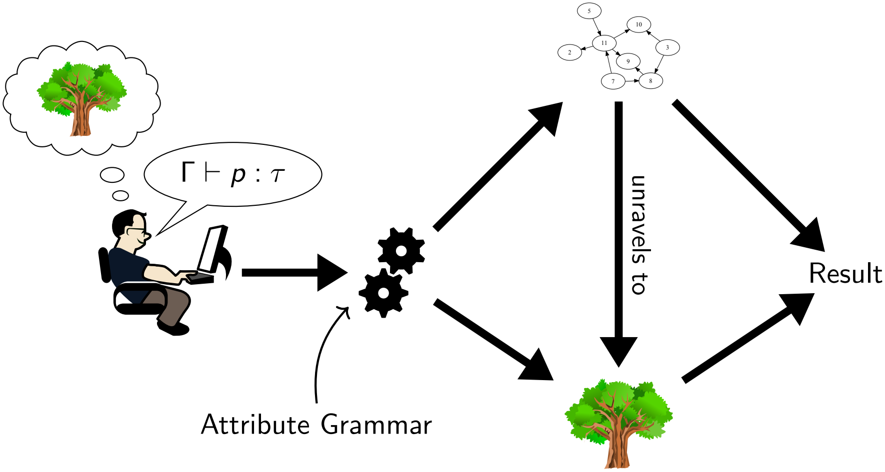
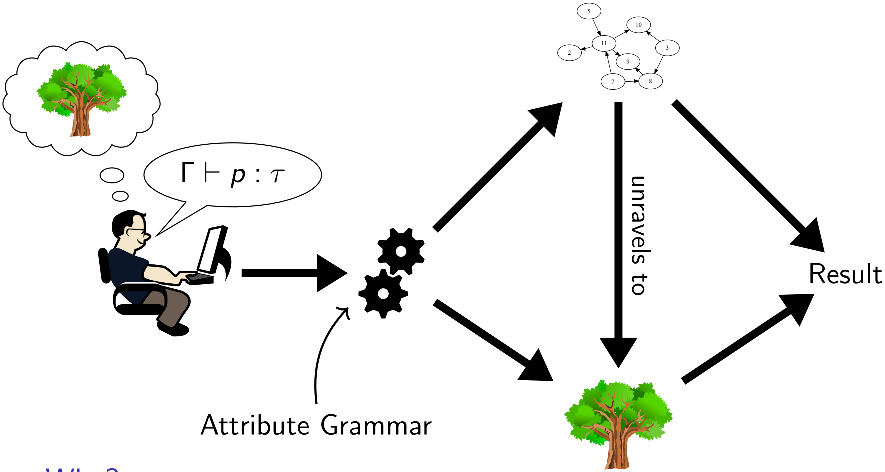


$\Gamma \vdash p : \tau$
5
10
11
2
3
9
7
8
unravels to
Result
Attribute Grammar

The Idea



Why?

- It's more difficult to get a traversal on graphs right.
- But: it's more efficient to traverse the graph.

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It doesn't work

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Our Contribution

- Identify classes of AGs for which this approach works.
- Prototype implementation in Haskell.
- Case studies and benchmarks.

A Toy Example

$$\begin{aligned}
\textbf{data}\ \mathit{IntTree} &= \mathit{Leaf}\ \mathit{Int} \\
&\mid \mathit{Node}\ \mathit{IntTree}\ \mathit{IntTree}
\end{aligned}$$

$$\begin{aligned}
&\mathit{leavesBelow} :: \mathit{Int} \to \mathit{IntTree} \to \mathit{Set}\ \mathit{Int} \\
&\mathit{leavesBelow}\ d\ (\mathit{Leaf}\ i) \\
&\quad \mid d \leqslant 0 \qquad\ = \mathit{Set.singleton}\ i \\
&\quad \mid \mathit{otherwise} = \mathit{Set.empty} \\
&\mathit{leavesBelow}\ d\ (\mathit{Node}\ t_1\ t_2) = \\
&\qquad\quad\ \ \mathit{leavesBelow}\ (d-1)\ t_1 \\
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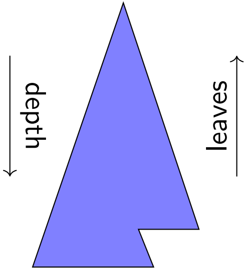

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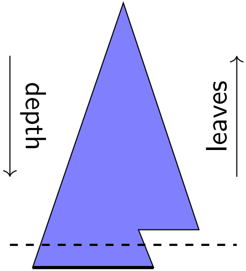
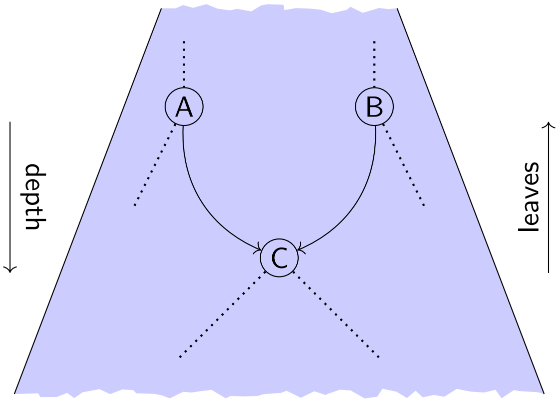


Traversal on Graphs



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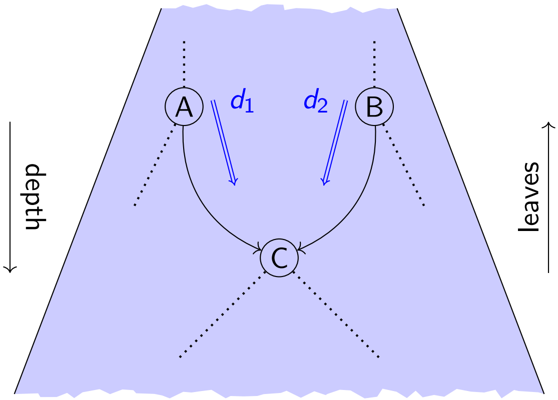

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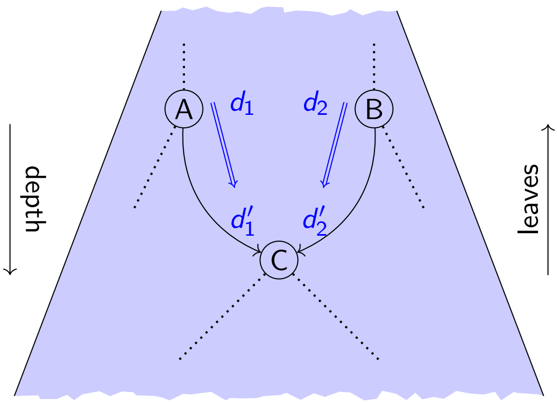

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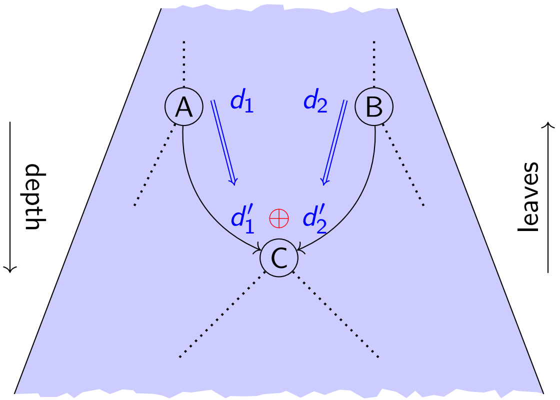

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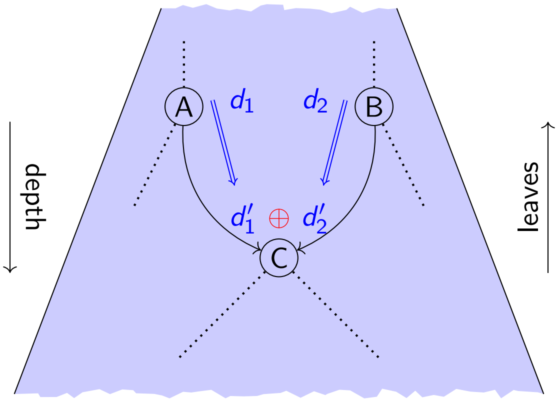

For which traversals is this correct?

But before that, let's implement it!

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\begin{aligned}
\textbf{data}\ \mathit{IntTreeF}\ a = {} & \mathit{Leaf}\ \mathit{Int} \\
\mid {} & \mathit{Node}\ a\ a
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$\oplus$

Implementing the semantic functions

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&\mathit{leavesBelow}_{\mathbf{I}} :: \mathit{Inh}\ \mathit{IntTreeF}\ \mathit{atts}\ \mathit{Int} \\
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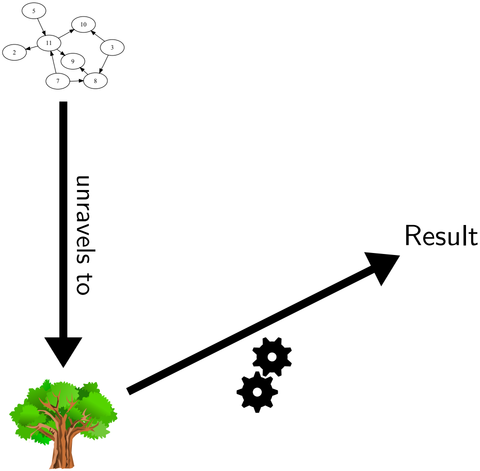
Correctness



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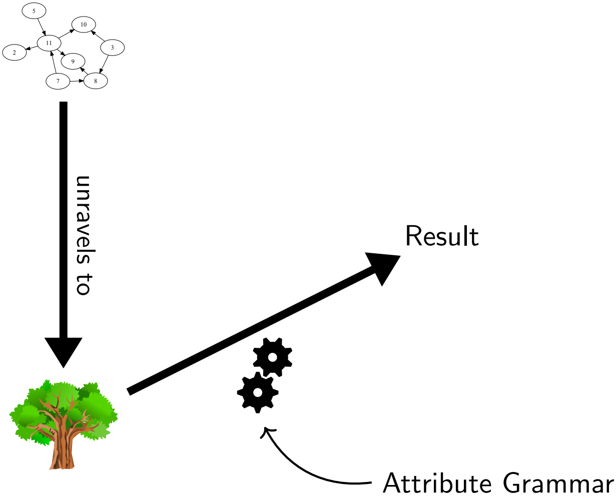

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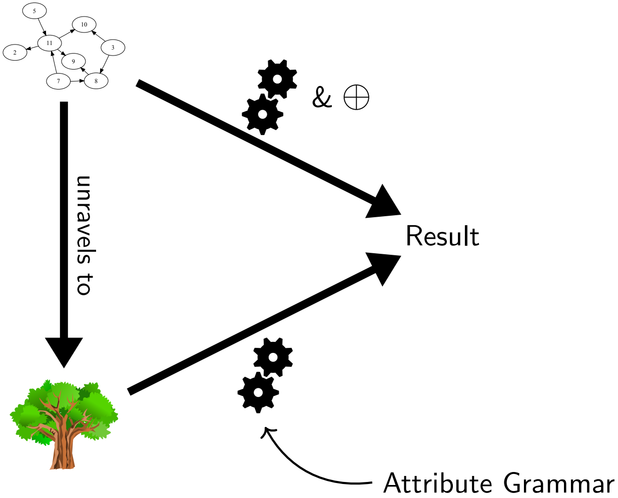

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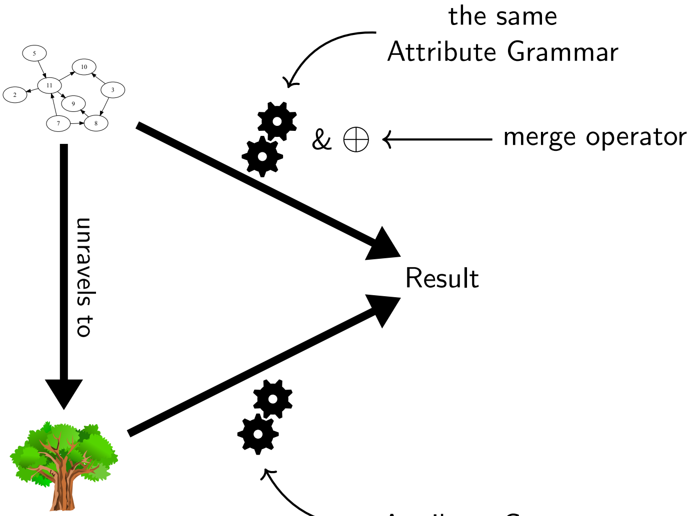

Correspondence Theorems

Theorem (Monotone AGs)

Let

(1) G *be a non-circular AG,*

(2) $\oplus$ *an assoc., comm. operator on inherited attributes, and*

(3) $\lesssim$ *such that* G *is monotone and* $\oplus$ *is decreasing w.r.t.* $\lesssim$*.*

If $(G, \oplus)$ *terminates on a DAG* g *with result* r*,*
then G *terminates on* $\mathcal{U}(g)$ *with result* r' *such that* $r \lesssim r'$*.*

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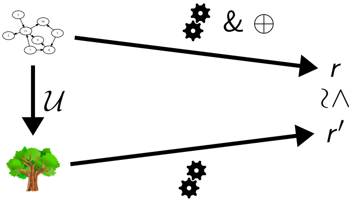

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Example

For the *leavesBelow* AG, define $\lesssim$ as follows:

- on *Int*: $x \lesssim y \iff x \leq y$
- on *Set Int*: $S \lesssim T \iff S \supseteq T$

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$$\implies \quad leavesBelow_G \; d \; g \supseteq leavesBelow \; d \; (\mathcal{U}(g))$$

for $leavesBelow_G \; d \; g \subseteq leavesBelow \; d \; (\mathcal{U}(g))$ see paper

Termination

- We know: non-circular AGs terminate on any tree.
- But: non-circular AGs may diverge on DAGs.

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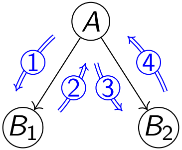

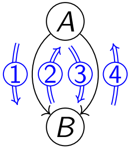

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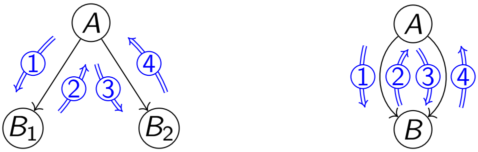

Theorem (termination)

Let G, $\oplus$, and $\lesssim$ be as before.

If $\lesssim$ is well-founded on inherited attributes, then $(G, \oplus)$ terminates on any DAG.

Correspondence Theorem for Copying AGs

Copying AGs

- inherited attributes are just propagated, not changed
- Example: Bird's repmin problem.

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Theorem (copying AGs)

Let

(1) *G be a copying, non-circular AG, and*

(2) *$x \oplus y \in \{x, y\}$ for all x, y.*

Then

(i) *$(G, \oplus)$ terminates on any DAG, and*

(ii) *$(\!|G, \oplus|\!)(g) = [\![G]\!](\mathcal{U}(g))$.*

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- Our framework generalises to tree/DAG-transformations
- Idea: attributes may contain trees/DAGs.

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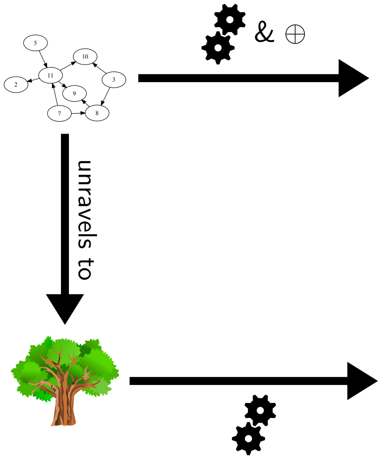

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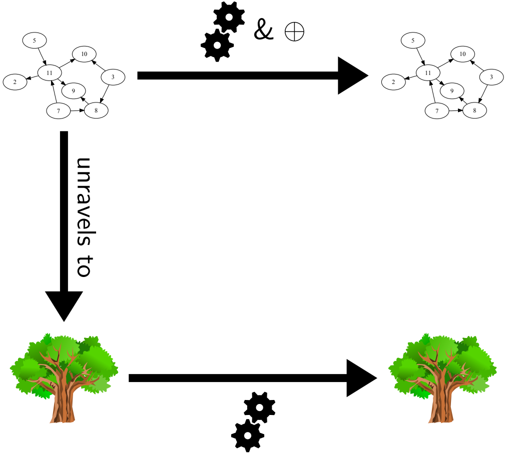

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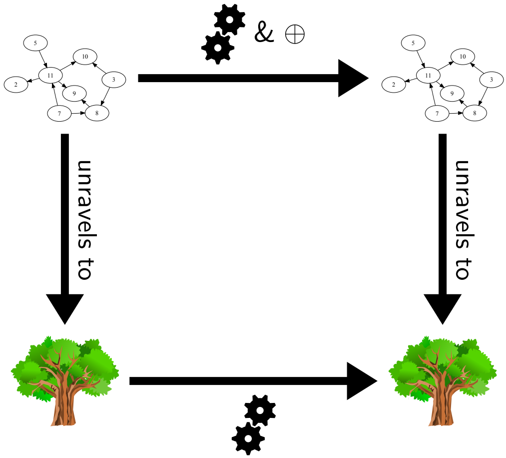

Example: Bird's Repmin Problem

$\textbf{newtype}\ Min_S = Min_S\ Int;$

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$$
\begin{aligned}
&min_S :: Syn\ IntTreeF\ atts\ Min_S\\
&min_S\ (Leaf\ i) \quad\ = Min_S\ i\\
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$$
\begin{aligned}
&repmin :: Tree\ IntTreeF \to Tree\ IntTreeF \\
&repmin = runRewrite\ min_S\ min_I\ rep\ init \\
&\quad \textbf{where}\ init\ (Min_S\ i) = Min_I\ i
\end{aligned}
$$

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&\mathit{repmin} = \mathit{runRewrite}\ \mathit{min}_S\ \mathit{min}_I\ \mathit{rep}\ \mathit{init} \\
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$$
\begin{aligned}
&\mathit{repmin}_G :: \mathit{Dag}\ \mathit{IntTreeF} \rightarrow \mathit{Dag}\ \mathit{IntTreeF} \\
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Summary

Our Contributions

- Haskell library to run AGs on DAGs
- Correspondence & termination theorems to prove correctness

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- Haskell library to run AGs on DAGs
- Correspondence & termination theorems to prove correctness

More in the paper

- Examples: type inference; circuits
- full theory & proofs
- parametric AGs ($\rightarrow$ tech report)
- Benchmarks ($\rightarrow$ tech report)

Conclusion

Future and Ongoing Work

- AGs with fixpoint iteration $\rightsquigarrow$ cyclic graphs
- mutually recursive data types and GADTs
- deep pattern matching in AGs
- corresponding notion of non-circularity for AGs on DAGs

Conclusion

Future and Ongoing Work

- AGs with fixpoint iteration $\rightsquigarrow$ cyclic graphs
- mutually recursive data types and GADTs
- deep pattern matching in AGs
- corresponding notion of non-circularity for AGs on DAGs

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Available from `http://j.mp/AG-DAG`.

- Haskell library source code
- more examples
- benchmarks

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Try the compositional datatypes library

```
> cabal install compdata-dags
```

Generalising Tree Traversals to DAGs

Exploiting Sharing without the Pain

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Source Code Repository

`http://j.mp/AG-DAG`

Haskell Library

```
> cabal install compdata-dags
```